Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Tuesday 10 June 2025

Afternoon (Time: 2 hours)

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 6 5 0 9 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The points *A*, *B*, *C* and *D* are the vertices of a quadrilateral *ABCD* such that

$$\overrightarrow{AB} = 3\mathbf{a} + 2\mathbf{b} \qquad \overrightarrow{AD} = 2\mathbf{a} - 5\mathbf{b} \qquad \overrightarrow{AC} = 5\mathbf{a} - 3\mathbf{b}$$

Show that *ABCD* is a parallelogram.

(4)

(Total for Question 1 is 4 marks)

2 The point P has coordinates $(-7, 3)$ and the point Q has coordinates $(3, 18)$

The point R divides the line PQ in the ratio $2:3$

The line l passes through R and is perpendicular to the line PQ

(a) Find an equation of line l
Give your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

(5)

The line l crosses the x-axis at the point S

(b) Find the area of triangle QRS

(5)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3

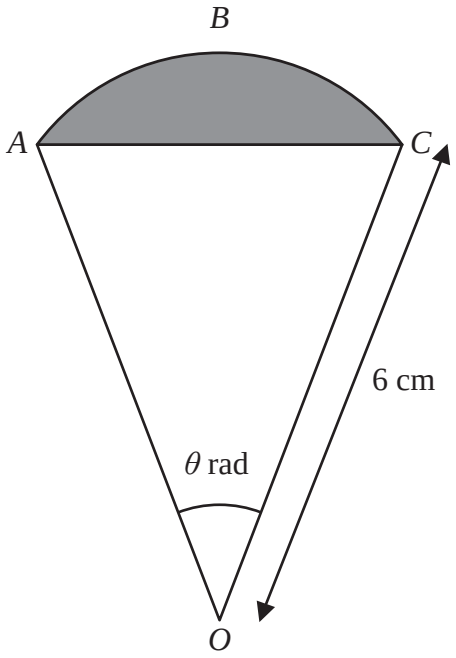

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows sector $OABC$ of a circle with centre O and radius 6 cm
The size of angle AOC is θ radians.

The perimeter of sector $OABC$ is $(12+\pi)$ cm

The region shown shaded in Figure 1 is bounded by the arc ABC and the line AC
The area of this region is S cm^2

Find the exact value of S

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4 (i) Find the value of $\sum_{k=6}^{17} 3(2)^{k-1}$

(4)

(ii) The common ratio of a geometric series G is positive.

The third term of G is $\frac{7}{13}$ and the ninth term of G is $\frac{448}{9477}$

G is convergent with sum to infinity S

Find the exact value of S

(7)

Question 4 continued

(Total for Question 4 is 11 marks)

5 $f(x) = x^3 + x^2 + x + c$ where c is a constant

The remainder when $f(x)$ is divided by $(x-2)$ is 4 times the remainder when $f(x)$ is divided by $(x+1)$

(a) Show that $c = 6$

(4)

Given that $(x+2)$ is a factor of $f(x)$

(b) show that the equation $f(x) = 0$ has only one real root.

(4)

Question 5 continued

(Total for Question 5 is 8 marks)

6

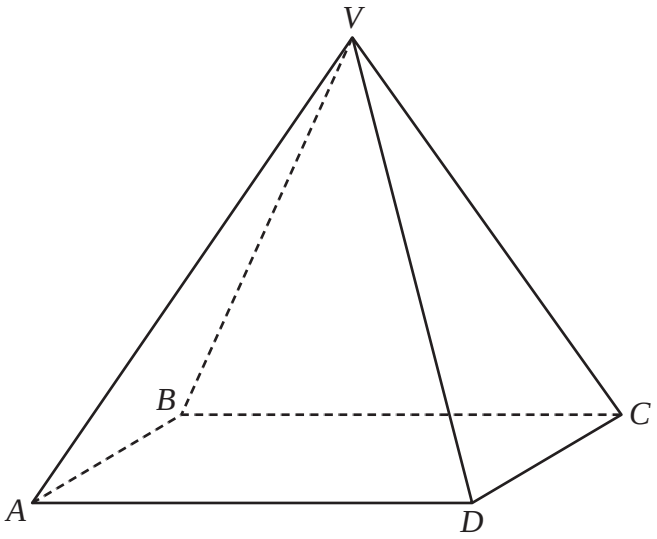

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Figure 2

Figure 2 shows a right pyramid *VABCD* with vertex *V* and rectangular base *ABCD*

$$AD = VC = 3AB$$

Find in degrees, to one decimal place, the size of the angle between the plane *CVD* and the base *ABCD*

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 6 marks)

7

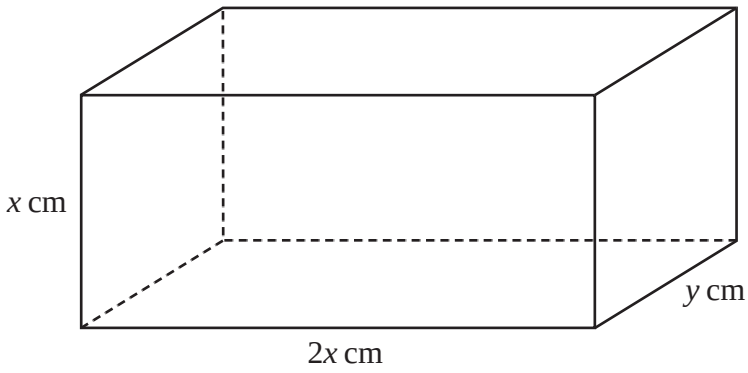

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Figure 3

Figure 3 shows a solid cuboid with sides of length x cm, $2x$ cm and y cm

The volume of the cuboid is 72 cm^3

The total surface area of the cuboid is A cm^2

(a) Show that $A = \frac{216}{x} + 4x^2$

(4)

(b) Use calculus to find the minimum value of A
Justify that your value is a minimum value of A

(7)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8 Use calculus to find the gradient of the curve with equation

$$y = e^{3x}(9x+2)^{\frac{1}{3}}$$

at the point on the curve where $x = 1$
Give your answer to 3 significant figures.

(5)

Question 8 continued

(Total for Question 8 is 5 marks)

9 (i) Solve the equation $3\left(\log_a 9 + \log_a 27\right) = 1$

Give your answer in the form $a = b^c$ where b is prime and c is an integer.

(3)

(ii) Solve the equation $\log_4 p + \log_p 256 = -4$

(6)

Question 9 continued

(Total for Question 9 is 9 marks)

10 The curve C has equation

$$y = \frac{5x - 2}{ax + b} \qquad x \neq -\frac{b}{a}$$

The asymptote to C that is parallel to the y-axis has equation $x = -\frac{4}{3}$

C crosses the y-axis at the point with coordinates $\left(0, -\frac{1}{4}\right)$

(a) (i) Show that $b = 8$

(ii) Find the value of a

(3)

(b) Write down the equation of the asymptote to C that is parallel to the x-axis.

(1)

(c) Find the coordinates of the point where C crosses the x-axis.

(1)

(d) Using calculus, show that at every point on C, the gradient is positive.

(4)

(e) Using the axes on the next page, sketch C
Clearly label the asymptotes and the coordinates where C crosses the coordinate axes.

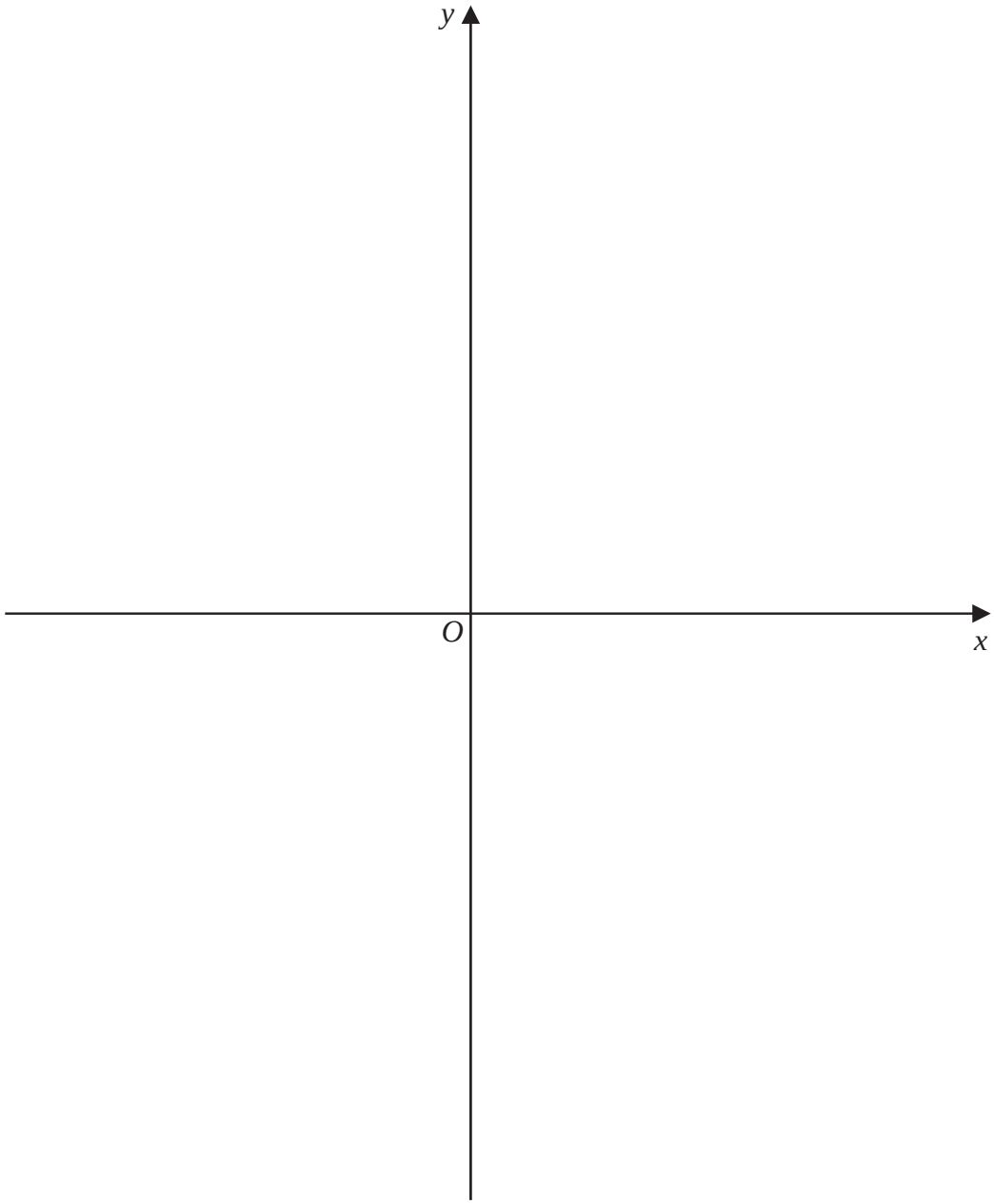
(3)

The gradient of C is $\frac{13}{100}$ at the point E and at the point F

(f) Find the length of the line EF, giving your answer to 2 decimal places.

(6)

Question 10 continued



Question 10 continued

Question 10 continued

(Total for Question 10 is 18 marks)

11

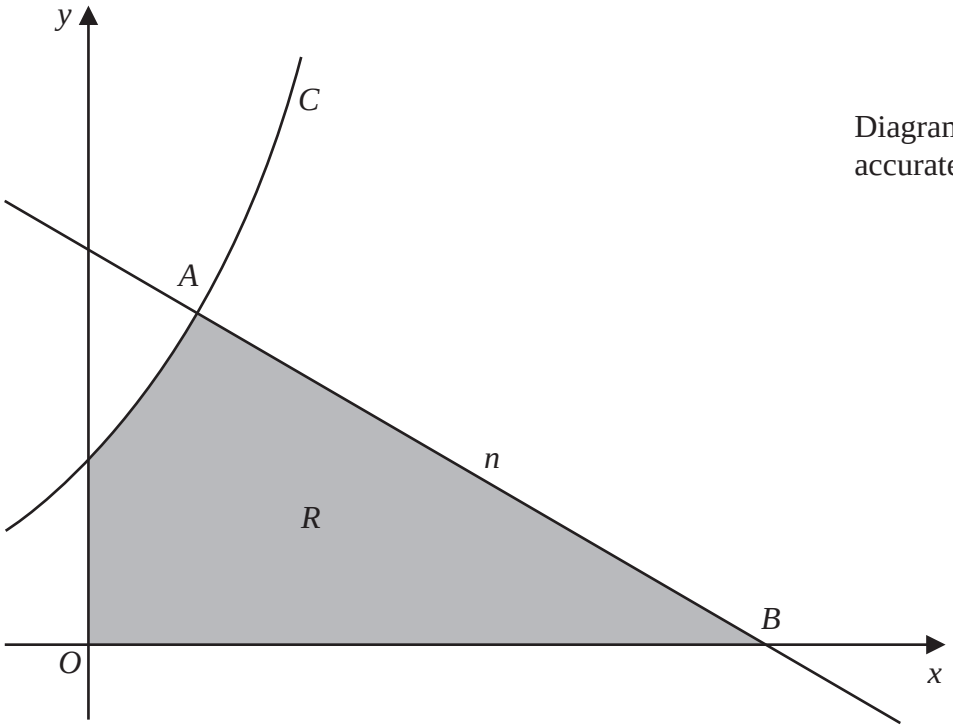


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows part of a curve C with equation $y = 3e^{\frac{x}{3}}$
The point A with coordinates $(2, p)$ lies on C

(a) Write down the exact value of p

(1)

The straight line n, shown on Figure 4, is the normal to C at A and crosses the x-axis at point B

The region R, shown shaded in Figure 4, is bounded by C, n, the x-axis and the y-axis.

(b) Use algebraic integration to find the area of R

Give your answer in the form $\frac{W}{V}e^2 + We^{\frac{2}{3}} - W$

where W and V are integers to be found.

(11)

Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS